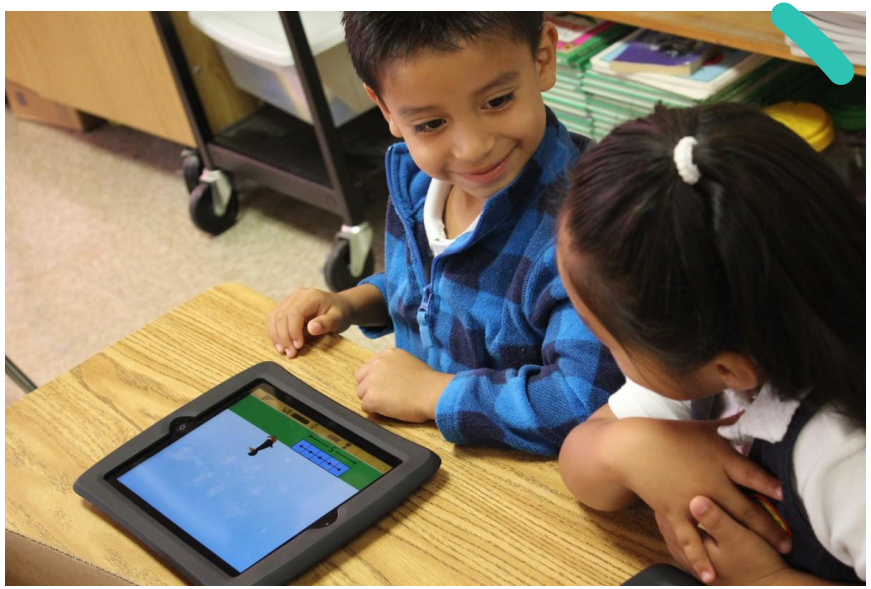
Evidence of Effective Math Instruction

Leanna Ferreira

Agenda

- 5 Strands of Mathematical Proficiency
- Effective Mathematics Teaching Practices (NCTM)
- Instruction Matters
- Connections to OTES 2.0



What defines effective math instruction for you?

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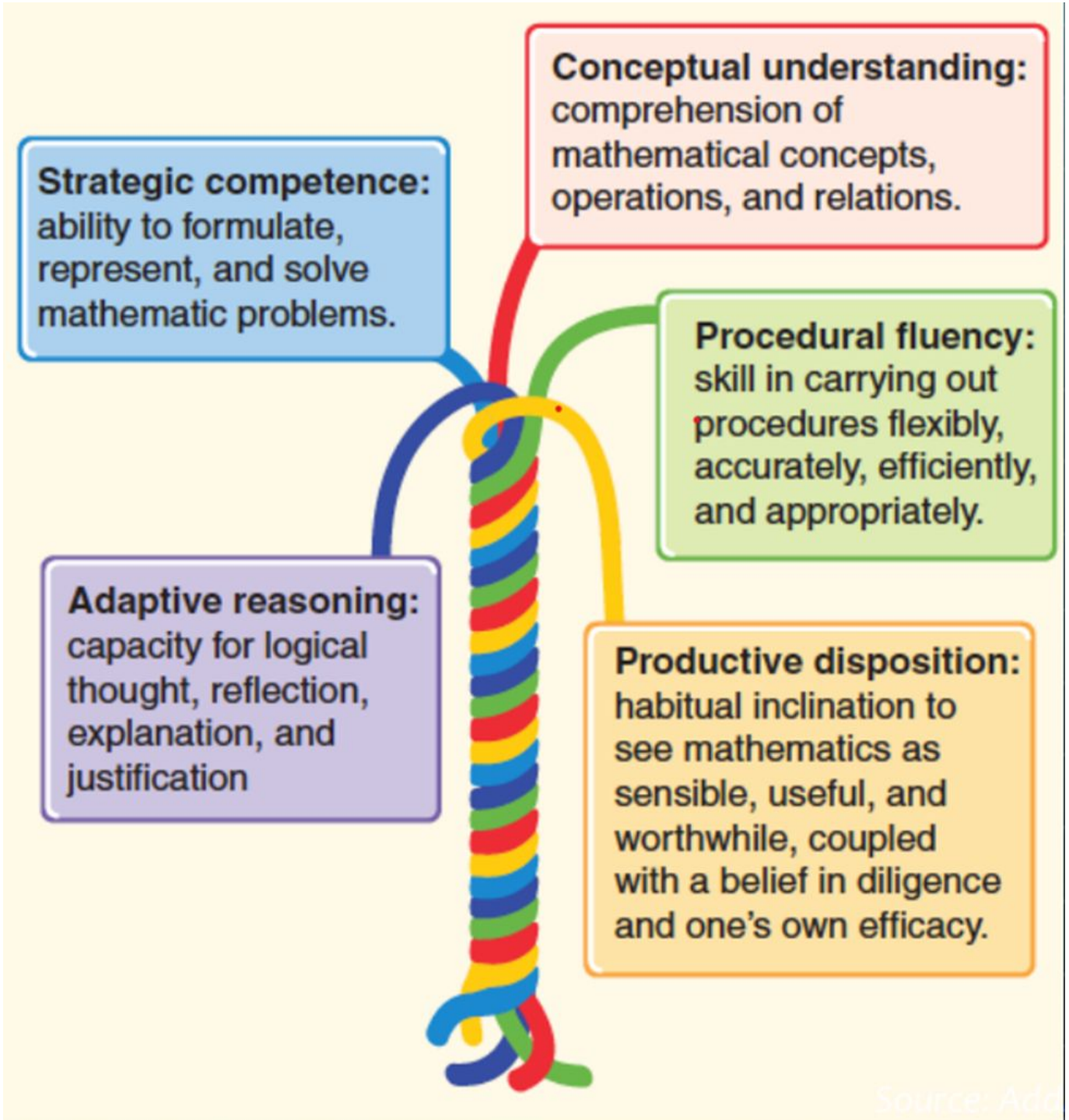
5 Strands of Mathematical Proficiency

Adding it Up


Conceptual understanding: comprehension of mathematical concepts, operations, and relations.
Strategic competence: ability to formulate, represent, and solve mathematic problems.
Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately.
Adaptive reasoning: capacity for logical thought, reflection, explanation, and justification
Productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Developing the Five Strands

Conceptual Understanding	• Enables students to connect ideas to what they already know • Supports retention and prevents common errors
Procedural Fluency	• Learning procedures can strengthen and develop mathematical understanding, while understanding makes it easier to learn skills
Strategic Competence	To come up with answer to a problem, students must: • follow a solution method and adapt as necessary • understand the quantities in the problem and their relationships • represent the relationships mathematically • have the mathematical skills required to solve the problem
Adaptive Reasoning	As students reason about a problem they can: • build their understanding • carry out the needed computations • apply their knowledge • explain their reasoning to others
Productive Disposition	Requires frequent opportunities to: • make sense of mathematics • recognize the benefits of perseverance • experience the rewards of sense making in mathematics

Effective Mathematics Teaching Practices

NCTM

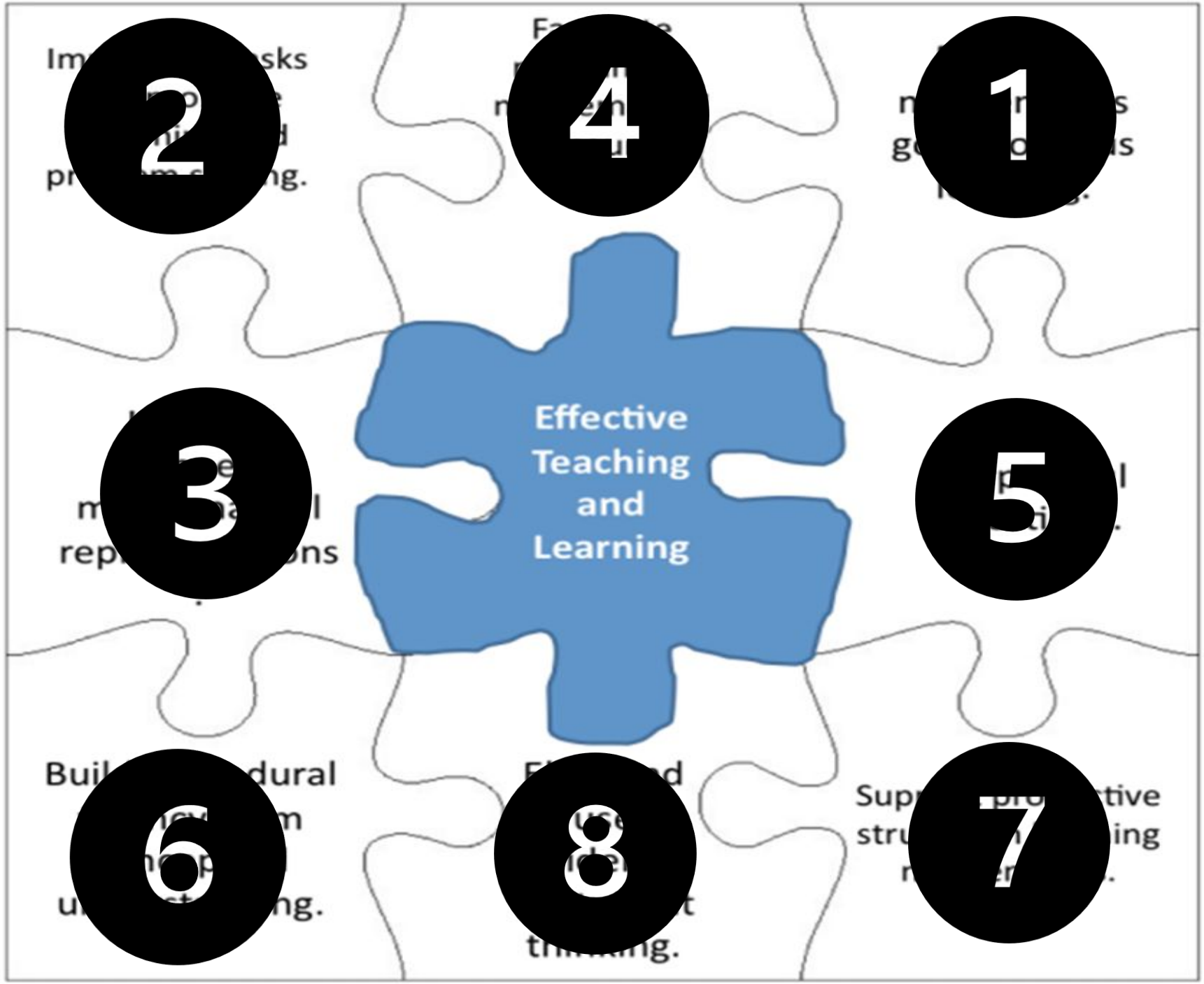
Effective Math Instruction

Encourages students to work cooperatively with others.

- Writing AND talking about math.
- Accepting divergent ideas.

Three critical components (Shellard & Moyer, 2002)

- Teaching for conceptual understanding.
- Developing children's procedural literacy.
- Promoting strategic competence through meaningful problem-solving investigations.



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K-2 Task

There are 10 cars in the parking lot. Some of the cars are red and some of the cars are black. How many red cards and how many black cars could be in the parking lot?

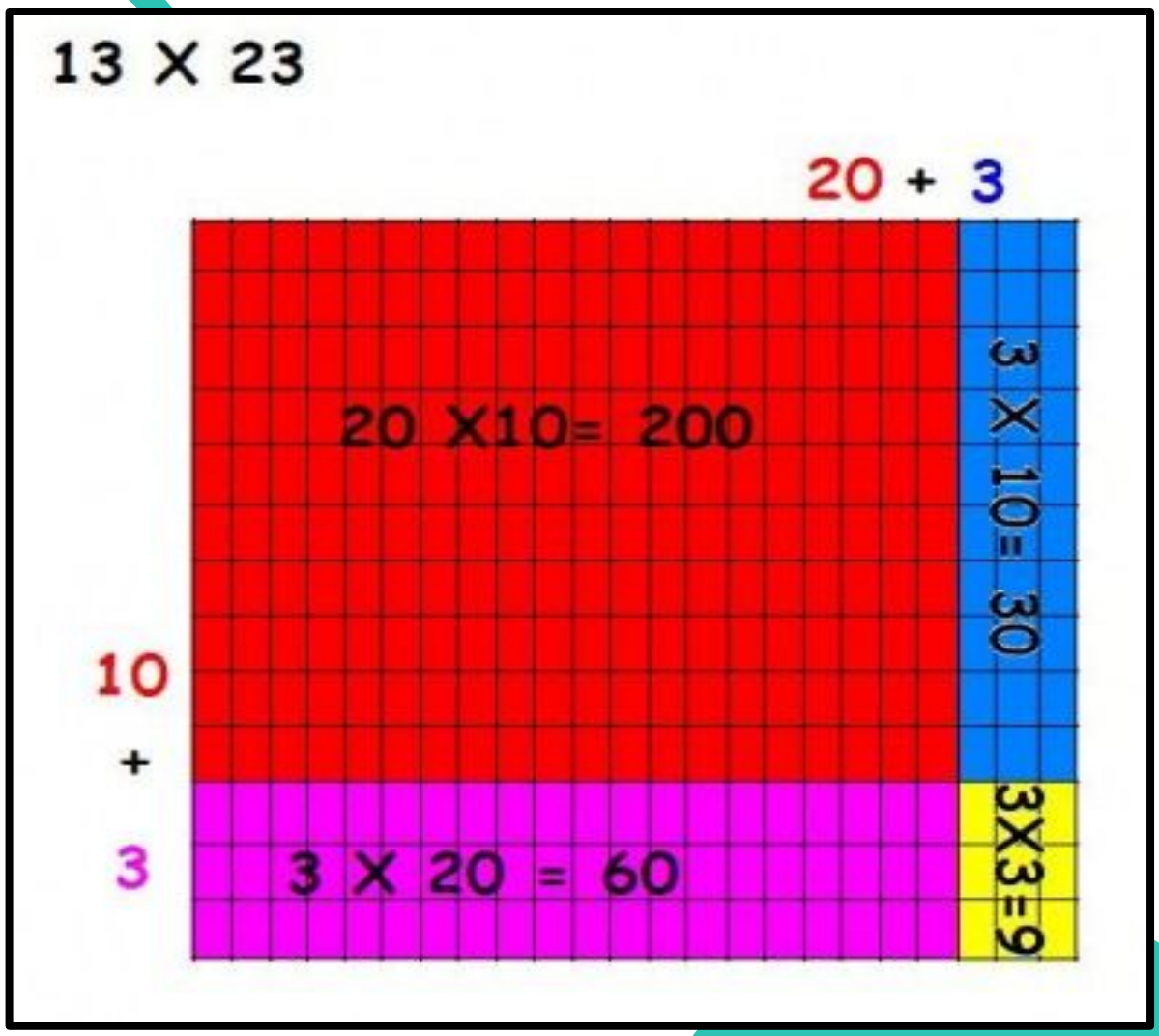
Think of as many different combinations of cars as you can.

Show your solutions in as many ways as you can with cubes, drawings or words, AND write an equation for each solution.

2-5 Task

Select a problem such as 23 x 13. Use base-ten blocks or grid paper to build the corresponding area model. Then, have students record and show as many ways as possible to “slice” the array into pieces.

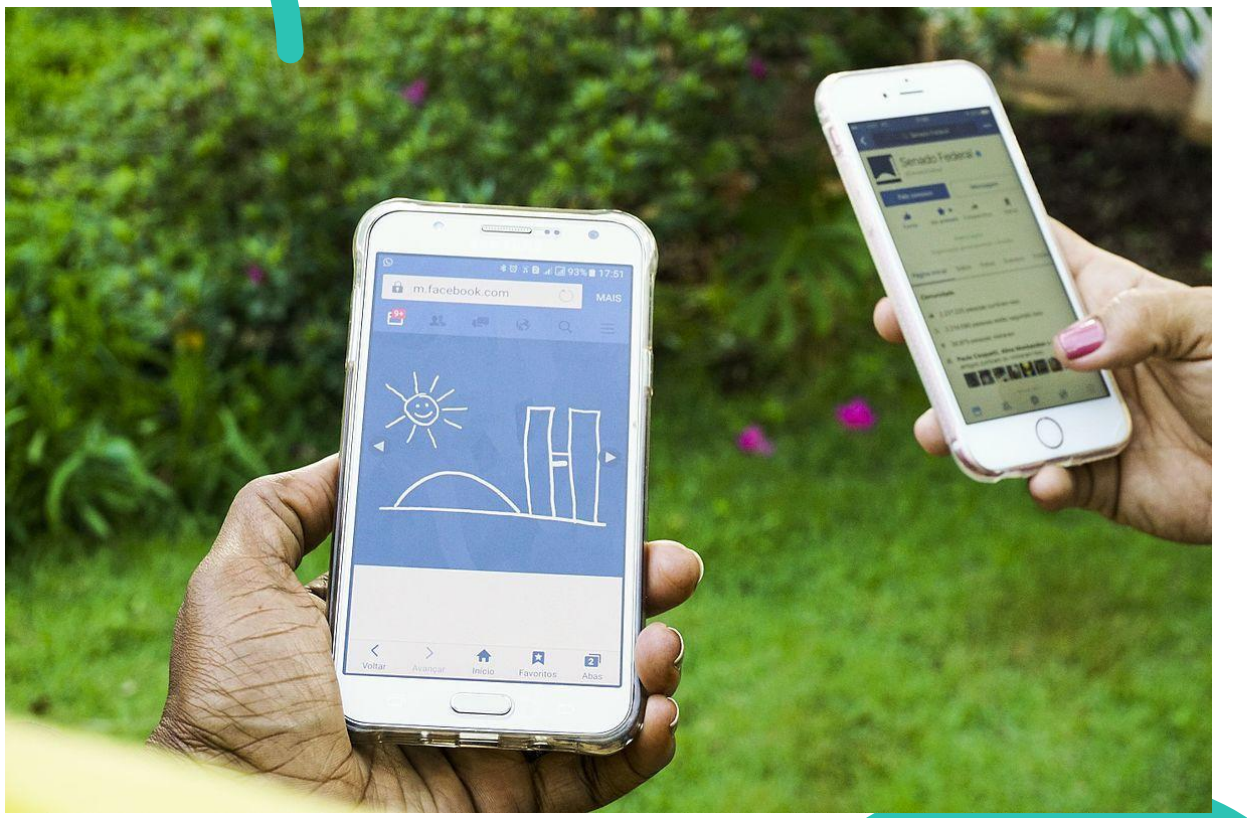
What other vertical or horizontal slices can be made? What property does this link to?



6-12 Task

You are trying to decide which of two smartphone plans would be better. Plan A charges a basic fee of $30 per month and 10 cents per text message. Plan B charges a basic fee of $50 per month and 5 cents per text message.

How many text messages would you need to send per month for plan B to be the better options? Explain your decision.



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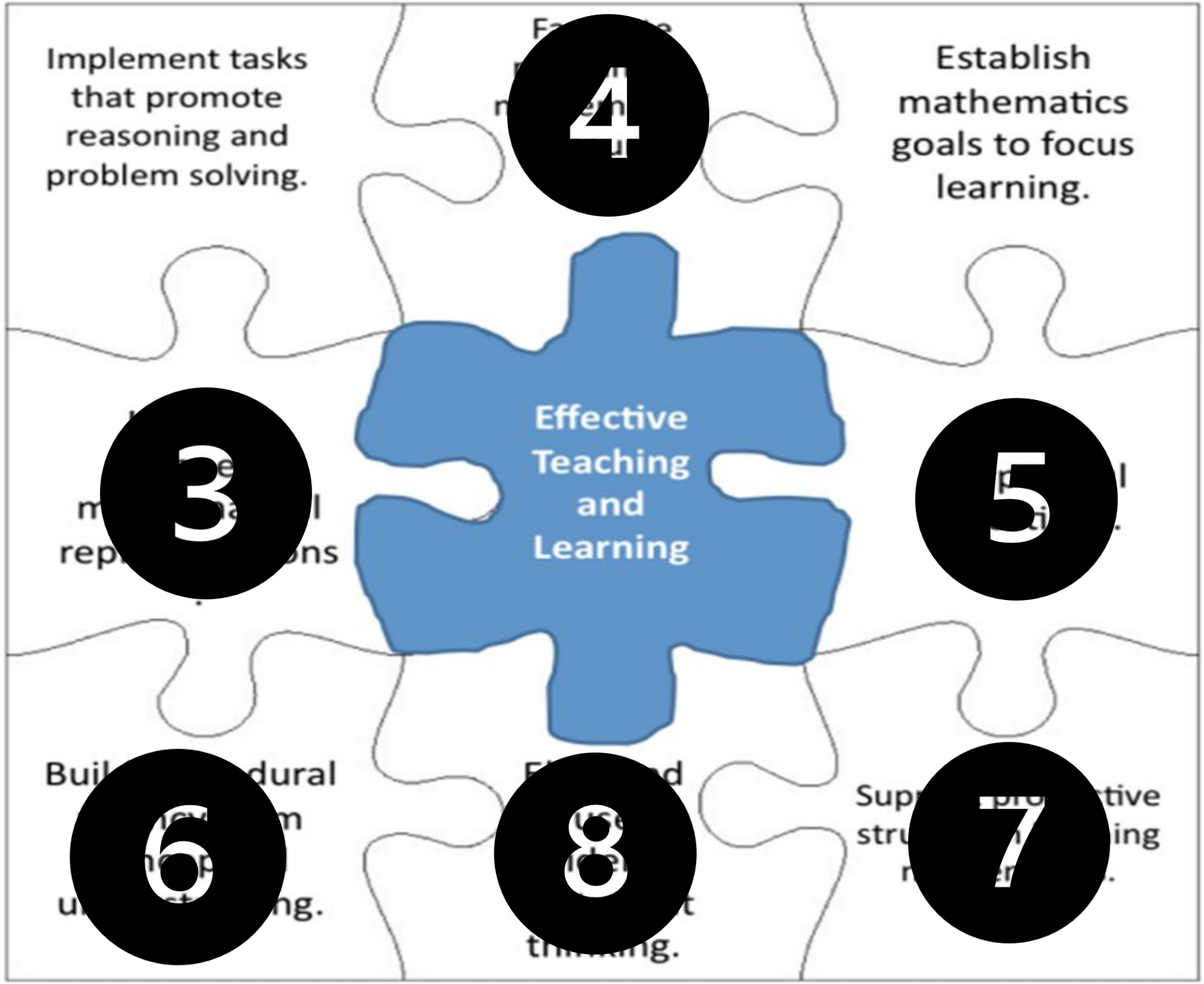

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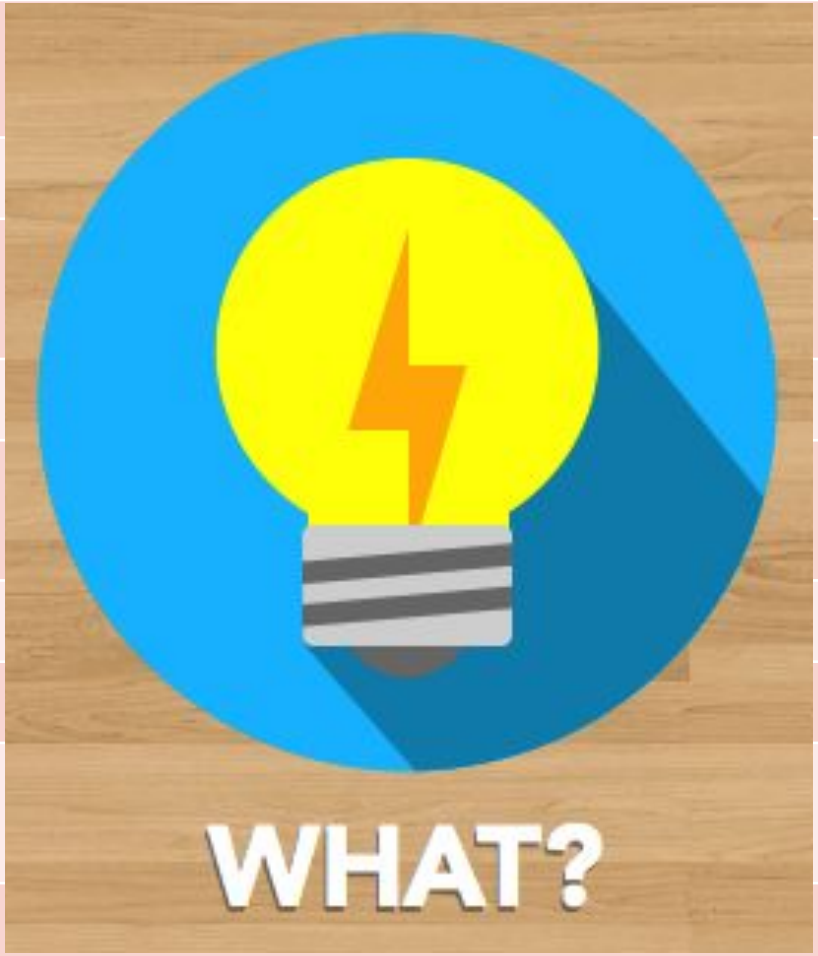
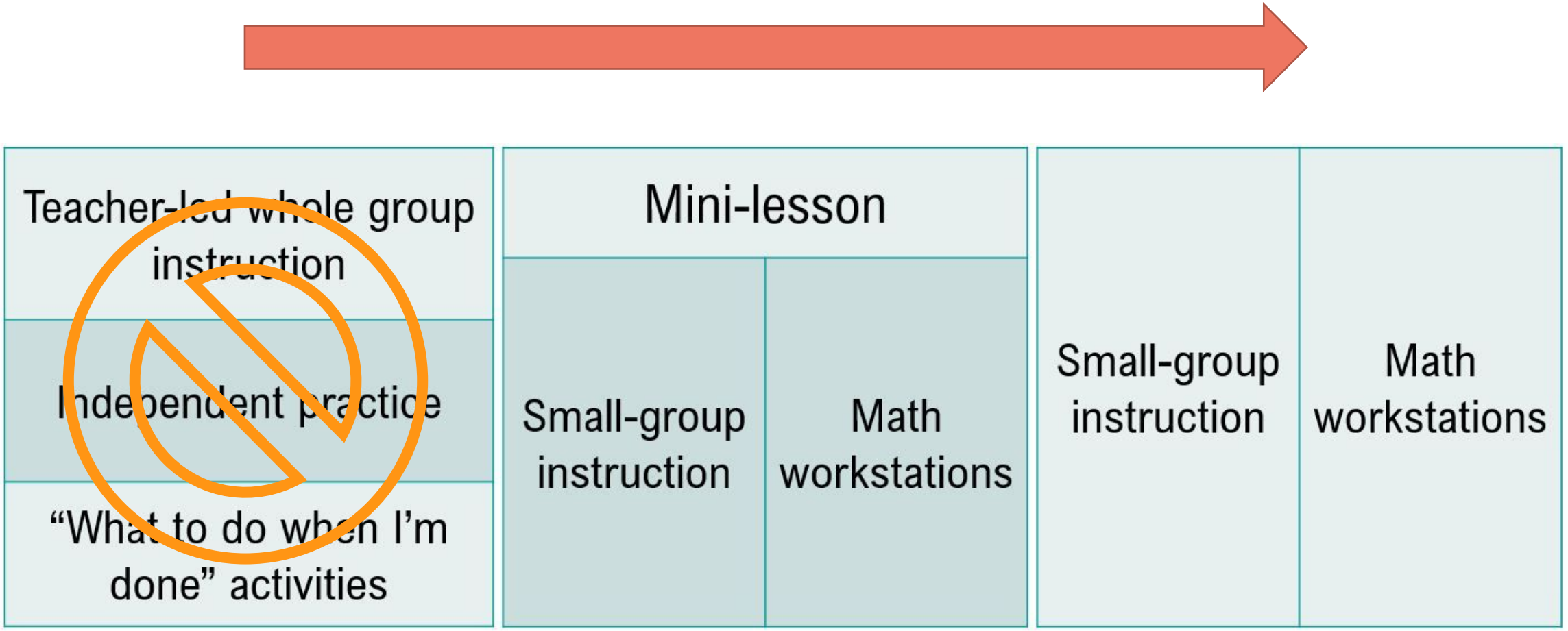
Connecting all three…

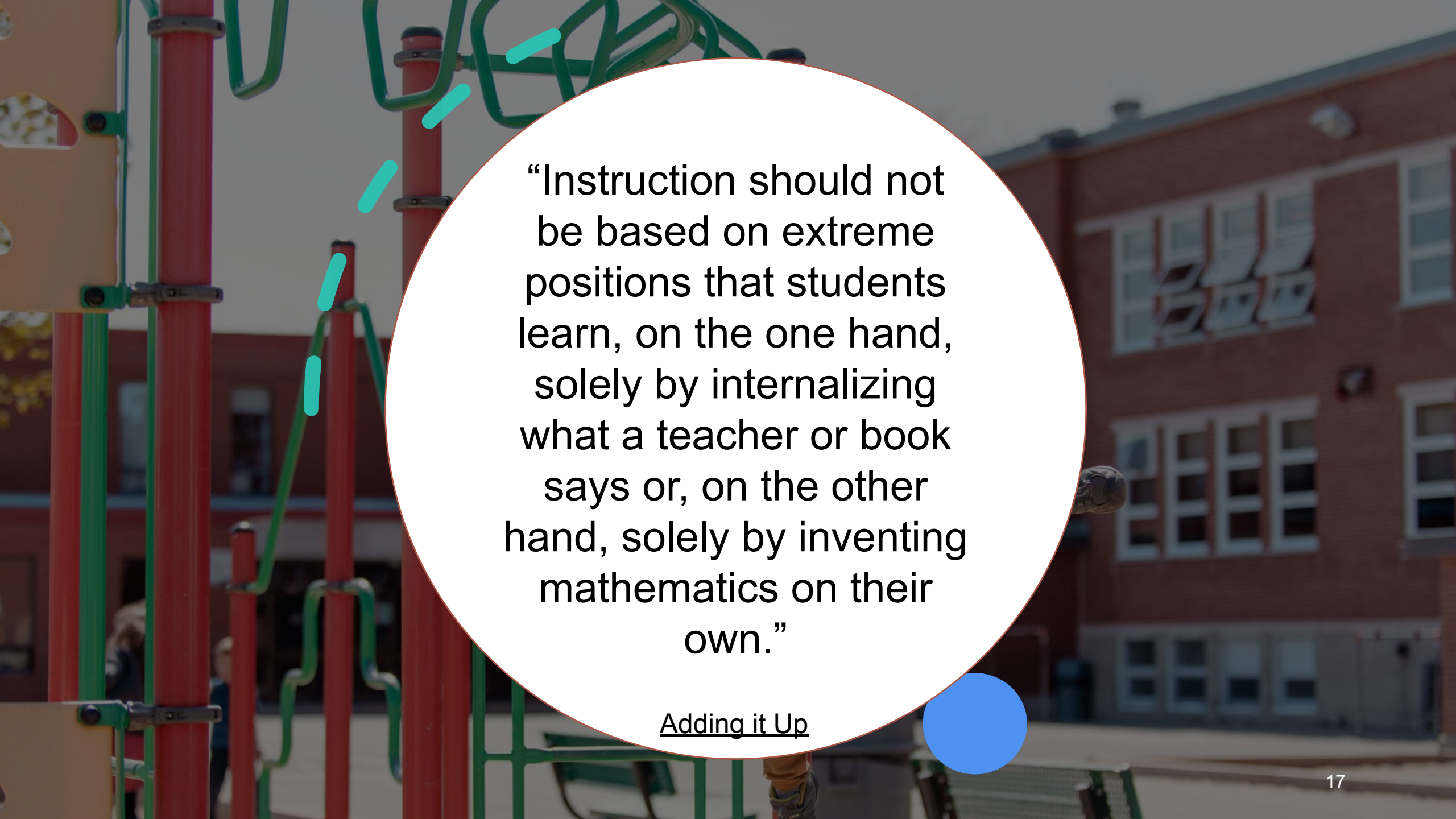
Effective Mathematics Teaching Practices	Standards for Mathematical Practice	5 Strands of Mathematical Proficiency
		

Instruction Matters

Research says…



Source: https://www.mathcoachscorner.com/2019/07/do-this-not-that-structuring-math-workshop-2/

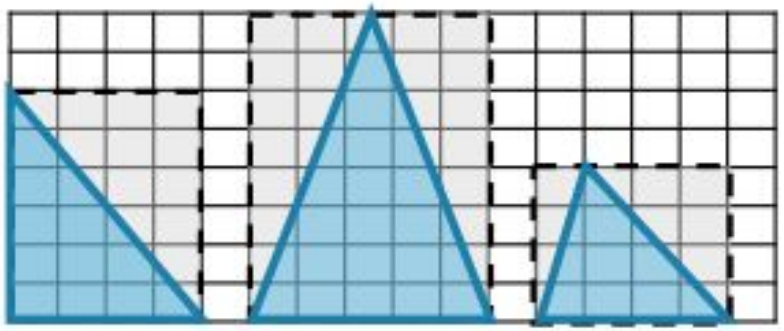


"Instruction should not be based on extreme positions that students learn, on the one hand, solely by internalizing what a teacher or book says or, on the other hand, solely by inventing mathematics on their own."

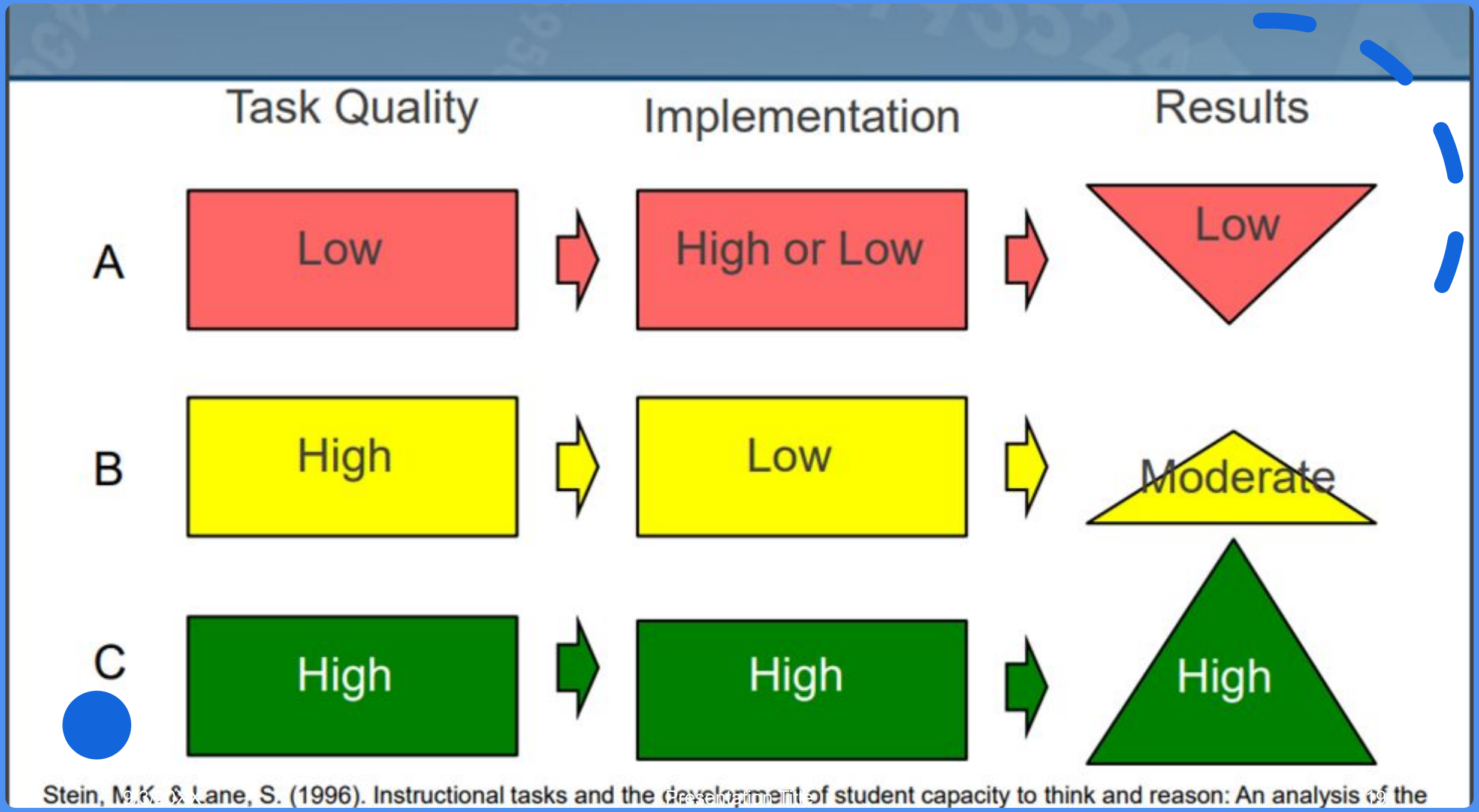
Adding it Up

Good Math Lesson

How can you determine the area of a triangle?



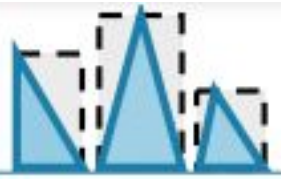
Explicit Instruction	Direct Instruction	Inquiry-based
Instruction focuses on critical content.	✓	✓
Break down complex skills and strategies.	✓	✓
Provide systematic instruction.	✓	✓
Provide judicious practice.	✓	✓
Require frequent responses.	✓	✓
Monitor student performance closely.	✓	✓
Provide immediate affirmation and corrective feedback.	✓	✓
Deliver the lesson at a brisk pace.	✓	✓
Help students organize knowledge.	✓	✓



Stein, M.K. & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the

Good Direct Instruction

How can you determine the area of a triangle?



For a warm-up the class solves rectangle area problems. We process solutions. With the handout on the document camera, I say to students:

- *The goal today is for you to learn how to determine the area of triangles and connect this to what you know about finding the areas of rectangles..*
- I have students determine the area of the triangle by counting squares. Next is whole class share. I verify with students the area by counting squares.
- Next I describe the circumscribed rectangle around this triangle and have students determine the area of the rectangle using the formula they know. Then whole class share and clarification.
- Next, I have students determine the areas of the second triangle and related rectangle. Then whole class share and verification of areas.
- I ask students to, "*Look back out our two rectangle-triangle sets? What do you notice about the areas?*" Students talk to a partner for a minute to share ideas.
- Then we have a whole class share. I suggest, confirm, and verify as needed that the triangles are half the areas of the related rectangles.
- I show the class how to write this as a formula. We discuss connections of the triangle area formula to the rectangle area formula, and connect back to the diagram.
- I have students use the formulas to determine the areas in a third set of diagrams and to verify the triangle area by counting the squares to see that it is reasonable.
- Assign another problem to practice. Process whole class. Check for understanding.
- Assign class a set of problems to practice upon.

Good Inquiry Lesson

How can you determine the area of a triangle?

For a warm-up the class solves rectangle area problems. We process solutions. With a copy of the handout on the document camera, I begin the lesson just as I did with the direct instruction model, going over the **first triangle – rectangle set**, with students doing much of the work as before, and I'm clarifying and correcting as needed.

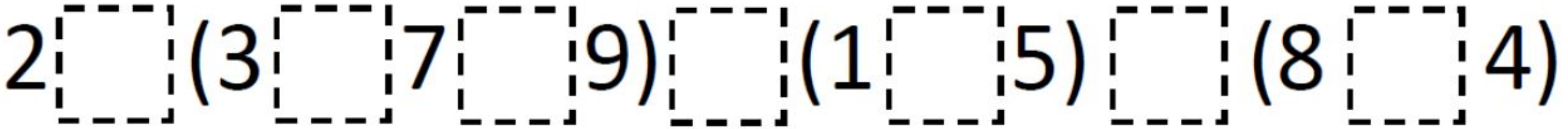
- Then I start the inquiry portion of the lesson. There are four more sets of triangles with circumscribed rectangles on cards in each group. Each group member is to take one, copy it onto graph paper, and determine the areas of the triangle and rectangle.
- After about 4 minutes students Rotate cards, draw their new triangle and rectangle on the graph paper and determine the areas.
- After another 4 minutes I stop the class and give them 2 minutes to compare areas with the group members sitting next to them. Then check these whole class.
- Tell students to, *"Look for patterns in the areas of the triangles and rectangles, and write a conjecture about the areas of the triangles."* (2-3 minutes, assist as needed).
- Then students share and discus their groups. Teacher picks 2-3 conjectures to share with the class. Connects the conjectures to the work, and guides the conjectures into a formula for finding the area of the triangle.
- *Turn and talk to a partner for 30-60 seconds about how the area formula the triangle relates to the area formula for a rectangle and why this makes sense.*
- Some students share whole class. Teacher clarifies, confirms, and connects, making sure a clear connection is stated for all students to hear. The remainder of the lesson is similar to the direct instruction model.

BALANCED EQUATIONS 2

Directions: Use the operation symbols (+, -, x, ÷) and equal sign (=) to make a true equation. Operation symbols may be used more than once.

What is the least value for each part of the equation? What is the greatest value for each part of the equation?

Can you complete the equation with at least one of each symbol?



Here is some information on each cookie:

Original Oreo

Nutrition facts per 3 cookies

Calories 160
Carbohydrate 25g
Protein 1g
Fat 7g
Weight 34 g

Double Stuf Oreo

Have about twice the normal amount of white creme filling.

Nutrition facts per 2 cookies

Calories 140
Carbohydrates 21
Protein < 1.0 g
Fat 7 g
Weight 29 g

Triple Double Oreo

Combine three wafers with two layers of creme, one vanilla, and one chocolate.

Nutrition facts per one cookie

Calories 100
Carbohydrate 15 g
Protein 0.5 g
Fat 4.5 g
Weight 21 g

Most Stuf

Nutrition facts per one cookie

Calories 110
Carbohydrates 14 g
Protein 0 g
Fat 5 g
Weight 21 g

1. Serving sizes are different for each type of cookie. How do you want to arrange the nutrition information so that you can easily compare these cookies?

For each of the bunnies below;

- Decide if you can tell whether the bunny is hollow or solid and explain how you know.
- Calculate each bunny's price per pound.
- Find each rabbit's price per inch tall.
- Where on your own body would this rabbit come to if you were standing beside it?

Solid or Hollow?				
	Lake Champlain Chocolates Burlington, VT $99.00 3 pounds More than 1 foot tall.	Reese's 1 pound Chocolate Bunny $29.00 Online at Amazon.com Looks about 11 inches tall.	Conrad Candies 26-inch tall Easter Bunny Weighs approximately 7lb. Hand molded and decorated hollow Milk Chocolate Easter Bunny. $191.98	Chocolate Easter Bunny named Topper from Dilettante Chocolates. Fine chocolate molded Easter Bunny stands 13 inches tall. Hand painted and exquisitely detailed. Made with three types of chocolate (white, dark, and milk) Total Net Wt. 40 oz $30.50

Beliefs about Teaching & Learning Mathematics

UNPRODUCTIVE

- Learning should focus on practicing procedures and memorizing basic number combinations.
- Students can learn to apply mathematics only after they have mastered the basic skills.
- An effective teacher makes the mathematics easy for students to building them step by step through problem solving to ensure they are not frustrated or confused.

PRODUCTIVE

- Learning should focus on developing understanding of concepts and procedures through problem solving, reasoning and discourse.
- Provide students with appropriate challenge, encourages perseverance in solving problems, and supports productive struggle in learning mathematics.
- Students can learn mathematics through exploring and solving contextual and mathematical problems.

PRODUCTIVE Beliefs about Teaching and Learning Mathematics

Mathematics learning should focus on developing understanding of concepts and procedures through problem solving, reasoning, and discourse.

All students need to have a range of strategies and approaches from which to choose in solving problems, including, but not limited to, general methods, standard algorithms, and procedures.

An effective teacher provides students with appropriate challenge, encourages perseverance in solving problems, and supports productive struggle in learning mathematics.

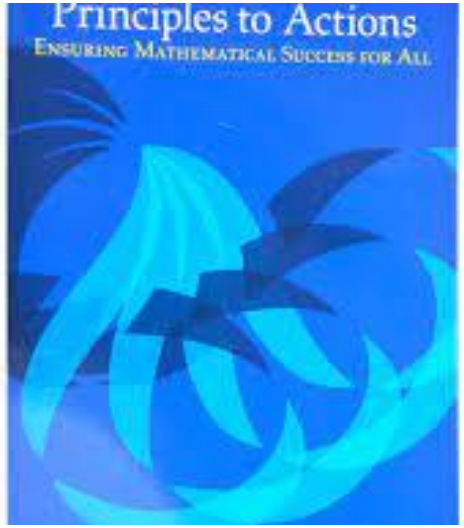
The role of the teacher is to engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves students toward shared understanding of mathematics.

The role of the student is to be actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and considering the reasoning of others.

Students can learn mathematics through exploring and solving contextual and mathematical problems.

OTES 2.0

Connections



Interaction…

- Explicit and practical steps to guide:
  - Instruction by teachers
  - Observations of learners and teachers
- Other ideas?

Thank you

Leanna Ferreira

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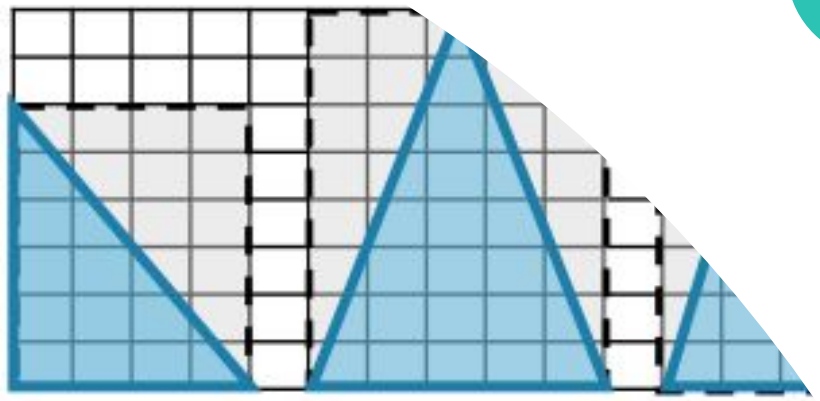
Happy to share these ideas with your staff this year. Contact Lynn Meister for details.

Poor Direct Instruction

How can you determine the area of a triangle?

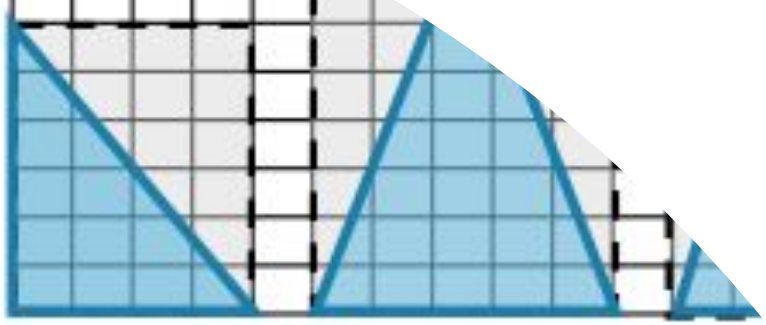


I say the following to students and diagram what I'm saying on the board:

- *The area of a triangle is computed by the formula, ½ (base x height).*
- *The base can be any side of the triangle you choose, and the height is an altitude, or line from the vertex opposite the base drawn directly to the base and perpendicular to the base.*
- *Here are some examples of computing the area of triangles….*(I show the examples, doing the math myself and having students take notes.)
- *Now turn to page xx in your books and compute the areas of the triangles on that page.*

Poor Inquiry Lesson

How can you determine the area of a triangle?



I put a copy of the handout (the three triangles on grid) on the document camera to show the class. I say to the class,

- *You have a paper with some triangles and rectangles drawn on them.*
- *Determine the areas of the rectangles and triangles and write about what you discover or any patterns you notice.*

After students work for a while I have students present their answers, acknowledging strategies for determining the area of the triangles, and assign additional work.

Success for All by…

- Engaging with challenging tasks.
- Connecting new learning with prior knowledge.
- Acquiring conceptual and procedural knowledge.
- Constructing knowledge through discourse.
- Receiving meaningful and timely feedback
- Developing metacognitive awareness.

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Task A: Percent Change Game

Use these 12 numbers to fill in the gaps below.

10, 20, 25, 35, 40, 50, 60, 70, 75, 80, 90, 100

$____ increased by ____% = $____

$____ increased by ____% = $____

$____ decreased by ____% = $____

$____ decreased by ____% = $____

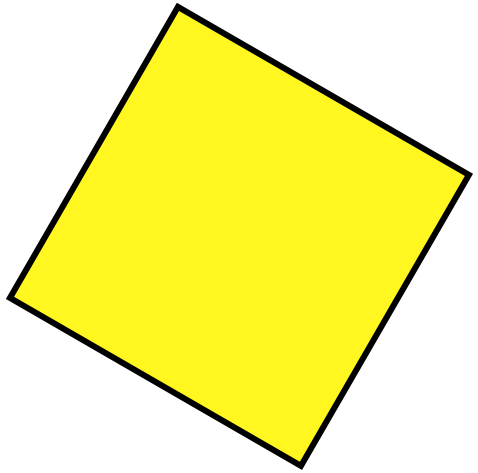
You score a point for each number **used only once** – and in a correct expression. (Maximum score: 12 points)

Adapted from http://www.foster77.co.uk/Percentage%20Change%20TASK%20SHEET.pdf

Task B: Describing and Defining a Square

Four equal sides

Diagonals meet at right angles



Two pairs of parallel sides

4 lines of symmetry

Two equal diagonals

Rotational symmetry of order 4

Four right angles

Which *pairs* of statements *define* a square?

Which pairs do not?

A lesson called 'Describing and Defining Quadrilaterals' based on this task, can be found on the Mathematics Assessment Project website (http://map.mathshell.org/)

Task C: Always, Sometimes or Never True?

If you add the same number to the top and bottom of a fraction, the fraction increases in value.

If you divide the top and bottom of a fraction by the same number, the fraction gets smaller in value.

If you multiply 12 by a number, the answer will be greater than 12.

If you divide 12 by a number, the answer will be less than 12.

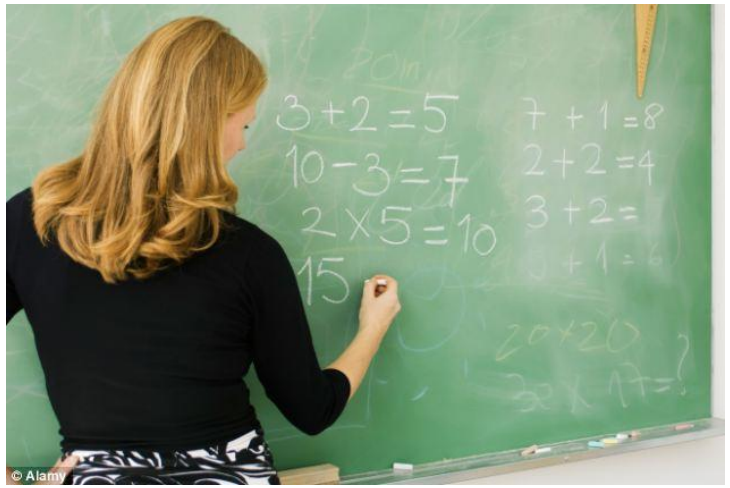
Prices increased by 20%.
They then decreased by 20%.
There was no overall change in prices.

Jill got a pay rise of 3%.
James got a pay rise of 2%.
Jill therefore got the greater pay rise.

Task D: Schoolteachers and Dentists

There are about 320 million people in the US.

- About how many school teachers are there?

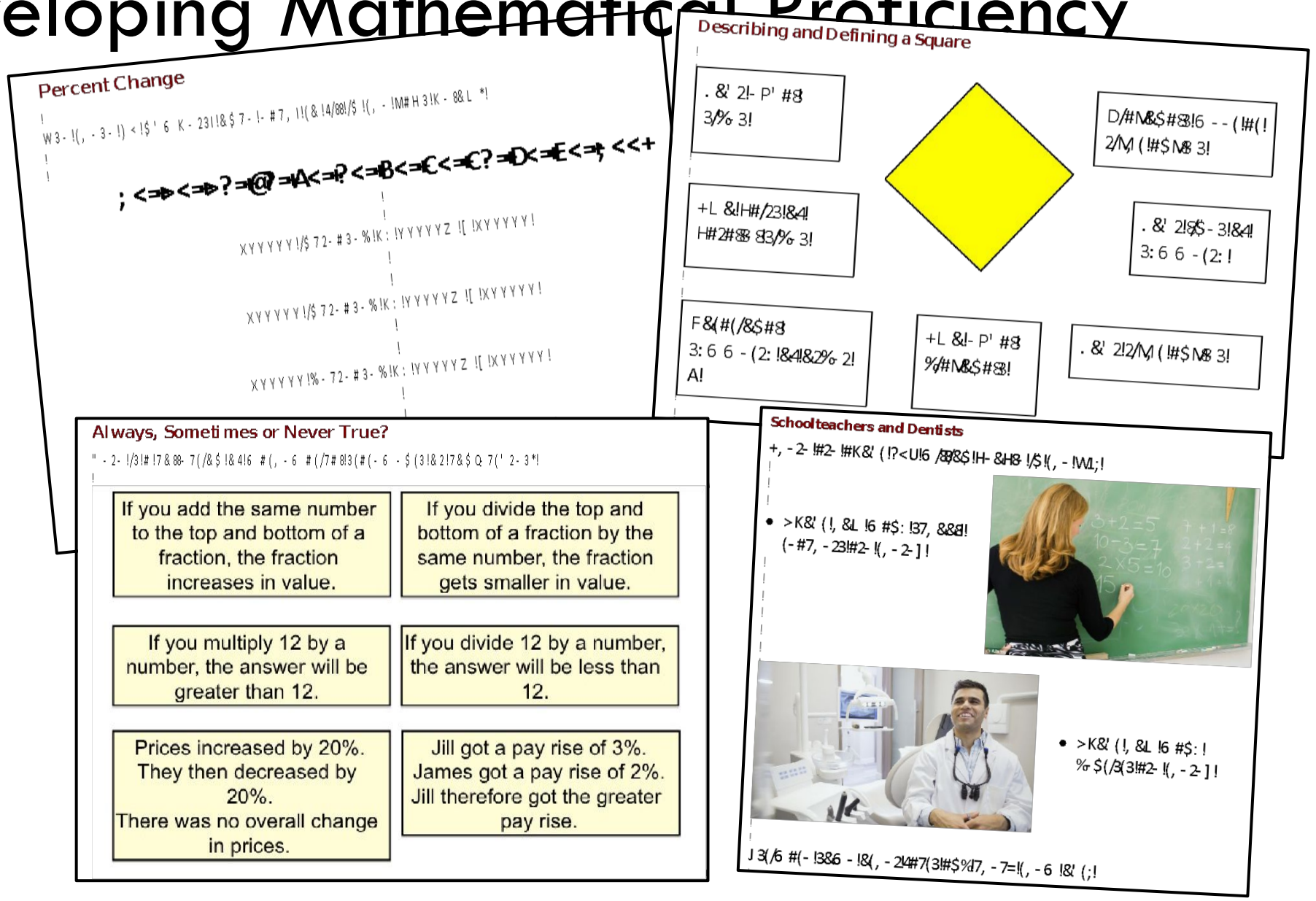


- About how many dentists are there?

Estimate some other facts and check them out.

Adapted from http://www.bowlandmaths.org.uk/pd/pd_01.html

Developing Mathematical Proficiency



Effective Mathematics Teaching Practices

- Establish mathematics goals to focus learning.
- Implement tasks that promote reasoning and problem solving.
- Use and connect mathematical representations.
- Facilitate meaningful mathematical discourse.
- Pose purposeful questions.
- Build procedural fluency from conceptual understanding.
- Support productive struggle in learning mathematics.
- Elicit and use evidence of student thinking.